

# Fractal and multifractal organization of neuroimaging signals in cognitive tasks and in disease

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<http://bionn.matinf.uj.edu.pl>

# Aims

- Differences between individuals/groups (healthy/diseased) for clinical diagnosis/biomarkers/prediction
- Differences in information processing
- ...detection aided by analysis of temporal patterns in neural signals

# Evidence for criticality

- **$1/f$  power spectra (fMRI, EEG, MEG)**

Zarahn E, Aguirre GK, D'Esposito M. Empirical analyses of BOLD fMRI statistics I. Spatially unsmoothed data collected under null hypothesis conditions. *NeuroImage* 1997; 5: 179–97.

Linkenkaer-Hansen K et al. Long-Range Temporal Correlations and Scaling Behavior in Human Brain Oscillations. *J Neurosci* 21(2001) 1370

- **Scaling size of neural activity  
(field potentials *in vivo/vitro*, fMRI)**

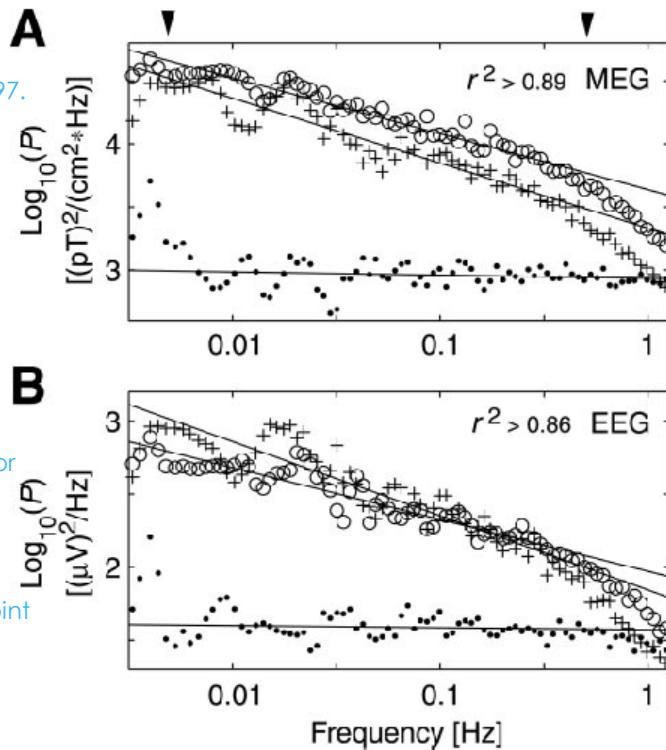
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- **Maximal susceptibility (fMRI)**

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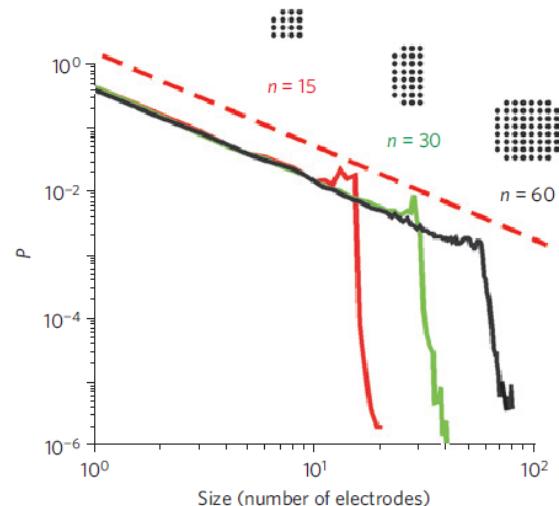
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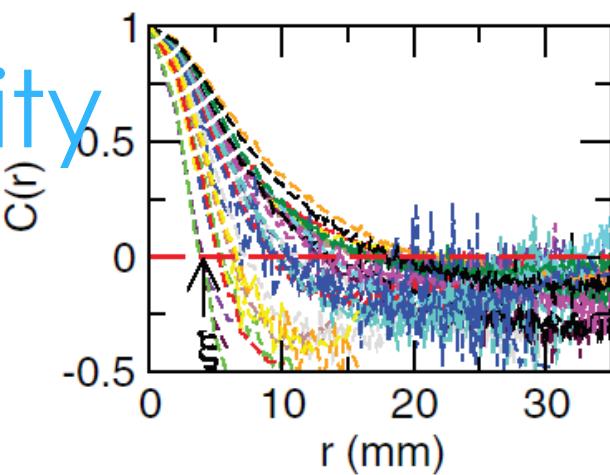
**Figure 1 |** Neuronal avalanches are complex. Size distribution of neuronal avalanches in mature cortical cultured networks follows a power law with an exponent close to  $3/2$  (dashed line) and exhibits finite-size scaling. The

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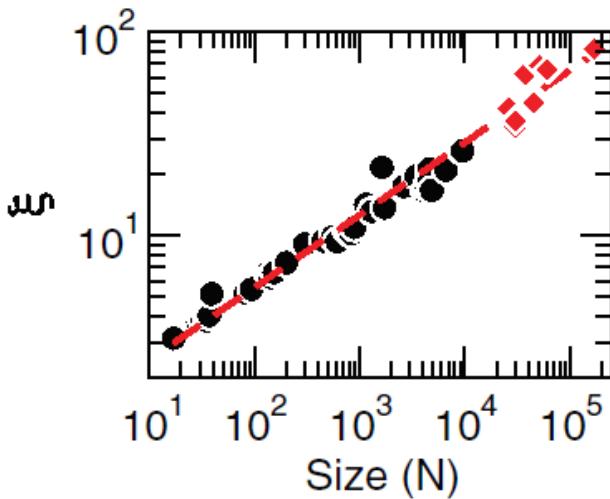
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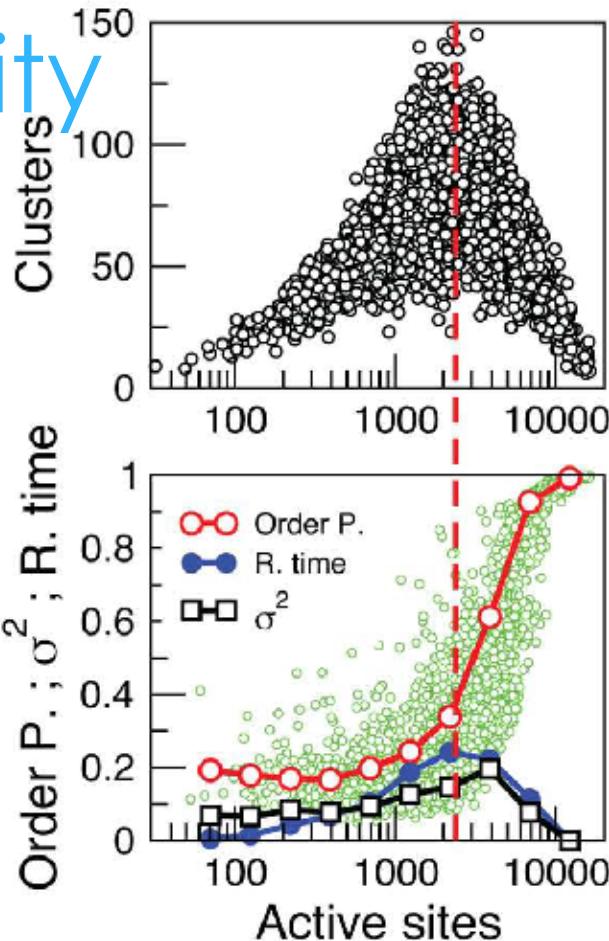
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# Non-trivial dynamics

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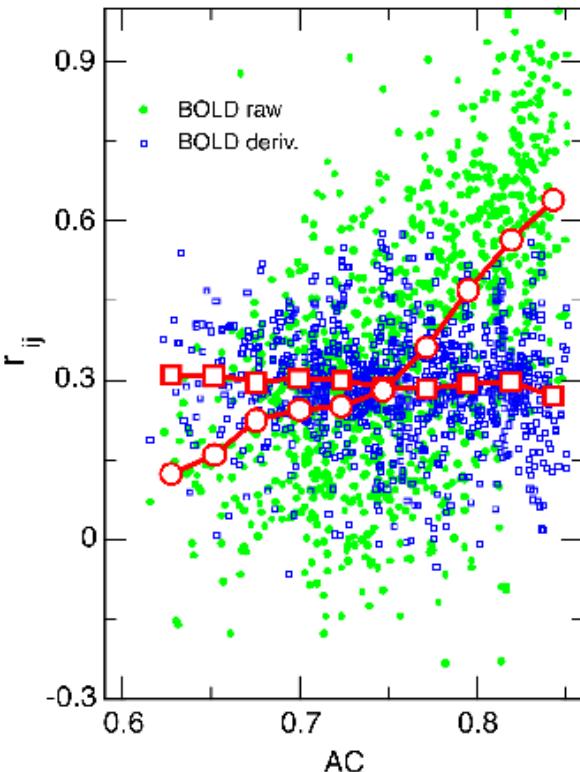
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# fMRI (functional magnetic resonance imaging )

Data: Working memory task

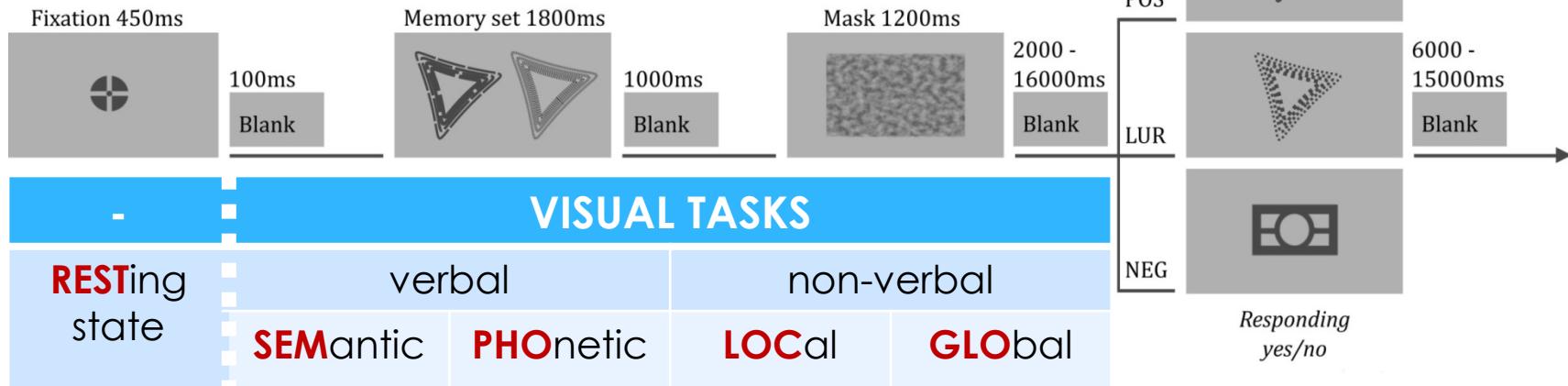
Technique: Hurst + detrended cross-correlations

# ENCoding

Was it in the memory set?

# REtrieval

Probe 2000ms



- 54 subjects (32 f., age:  $24.17 \pm 3.56$ ) selected with Pittsburgh Sleep Quality Index, Epworth Sleepiness Scale and genetic testing for the polymorphism of clock gene PER3, and divided into 26 morning-oriented and 28 evening-oriented types
- 2 sessions of modified DRM paradigm task: morning and evening

Lewandowska et al. Would you say “yes” in the evening? Time-of-day effect on response bias in four types of working memory recognition tasks. Chronobiol. Int. 35, 80–89 (2018).

J Deese (1959). Journal of Experimental Psychology, 58(1):17–22

HL Roediger and KB McDermott (1995). Journal of Experimental Psychology: Learning, Memory, and Cognition, 21(4):803–814

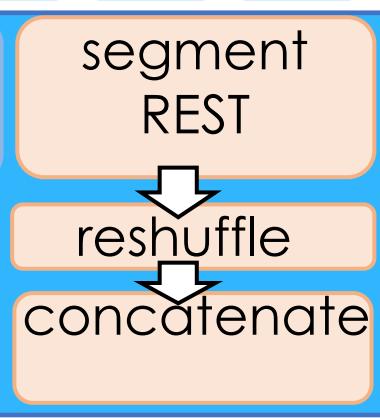
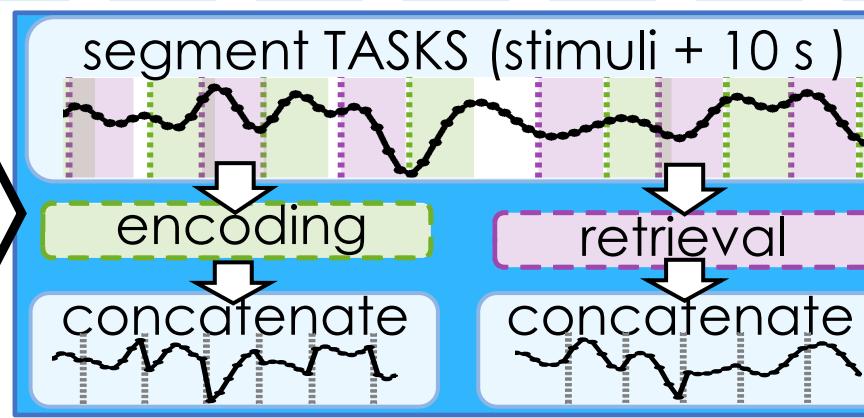
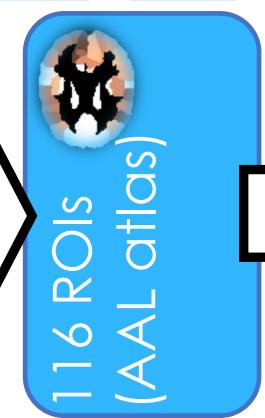
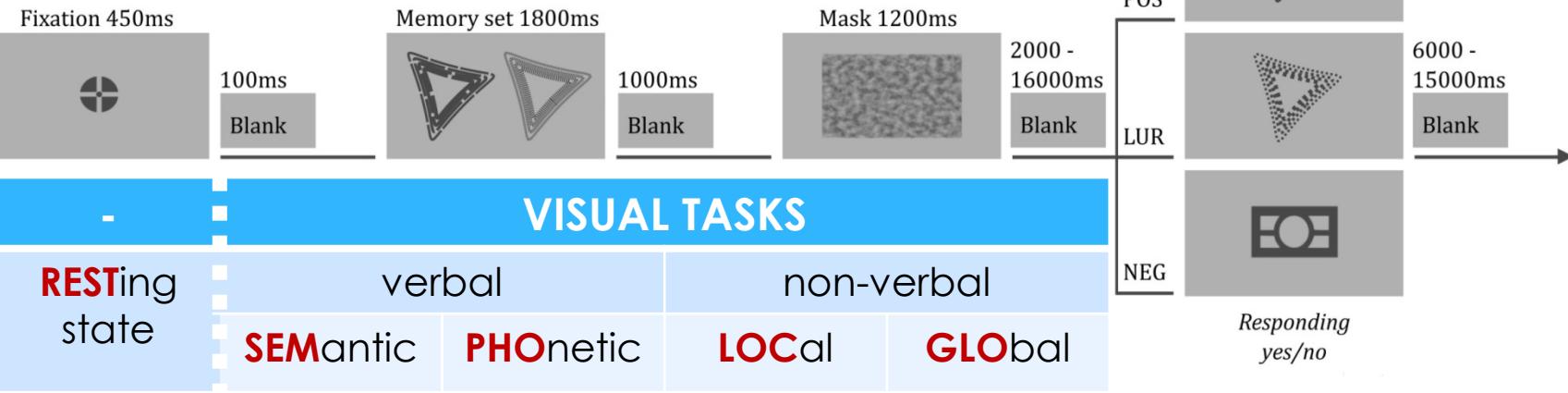
AS Atkins and PA Reuter-Lorenz (2011). NeuroImage, 56(3):1726–1734

# ENCoding

Was it in the memory set?

# REtrieval

Probe 2000ms



# Detrended fluctuation analysis

Take a time series

$$X(j) = \sum_{i=1}^j [x_i - \langle x \rangle]$$

detrend and sum MSE

$$F^2(\nu, s) = \frac{1}{s} \sum_{k=1}^s [X((\nu-1)s+k) - P_\nu(k)]^2$$

average fluctuation in segments

$$F(s) = \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} F^2(\nu, s) \right\}^{1/2}$$

get Hurst exponent

$$F(s) \sim s^H$$

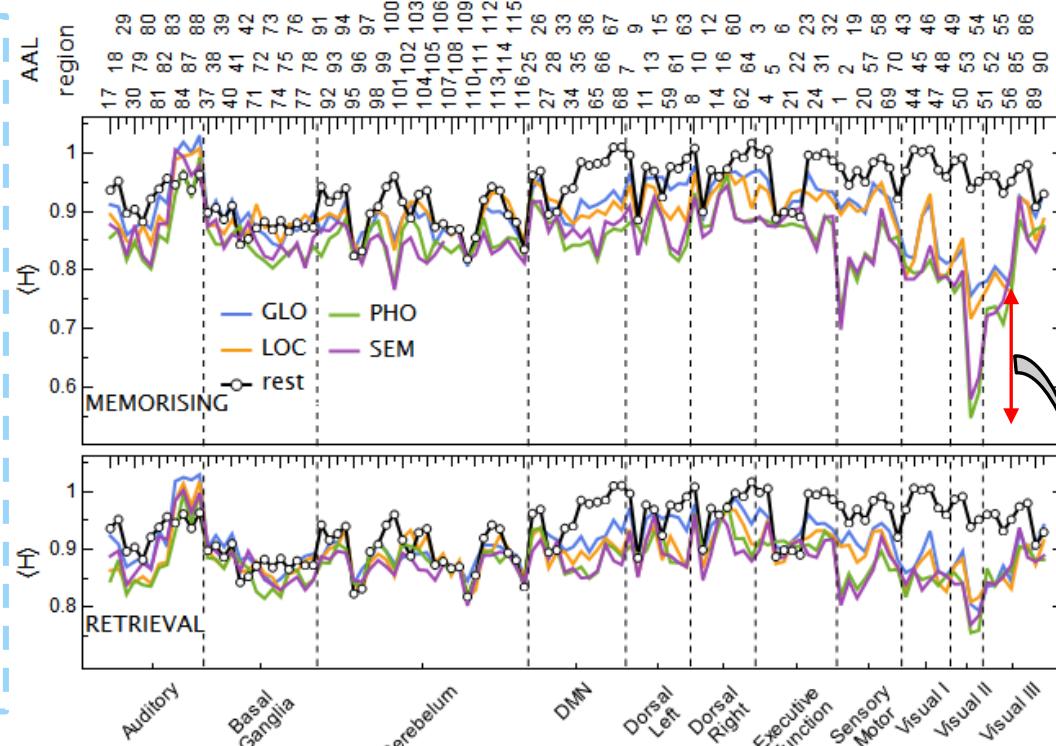
short-range correlated:

$$H = 0.5$$

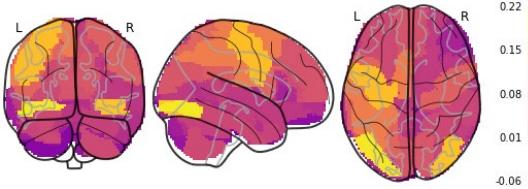
long-range monofractal:

$$0 < H < 0.5 \text{ (antipersistent)}$$

$$0.5 < H < 1 \text{ (persistent)}$$



persistent (temporally correlated)  
behaviour differences in specific  
areas between conditions



C-K Peng et al. (1994). Physical review E, 49(2):1685

P Oświęcimka et al. (2006). Physical Review E 74, 016103

# Detrended cross-correlation

Take 2 time series

$$X(j) = \sum_{i=1}^j [x_i - \langle x \rangle] \quad Y(j) = \sum_{i=1}^j [y_i - \langle y \rangle]$$

detrend and sum MSE

$$F_{XY}^2(\nu, s) = \frac{1}{s} \sum_{k=1}^s [X((\nu-1)s+k) - P_{X,\nu}(k)] \times [Y((\nu-1)s+k) - P_{Y,\nu}(k)]$$

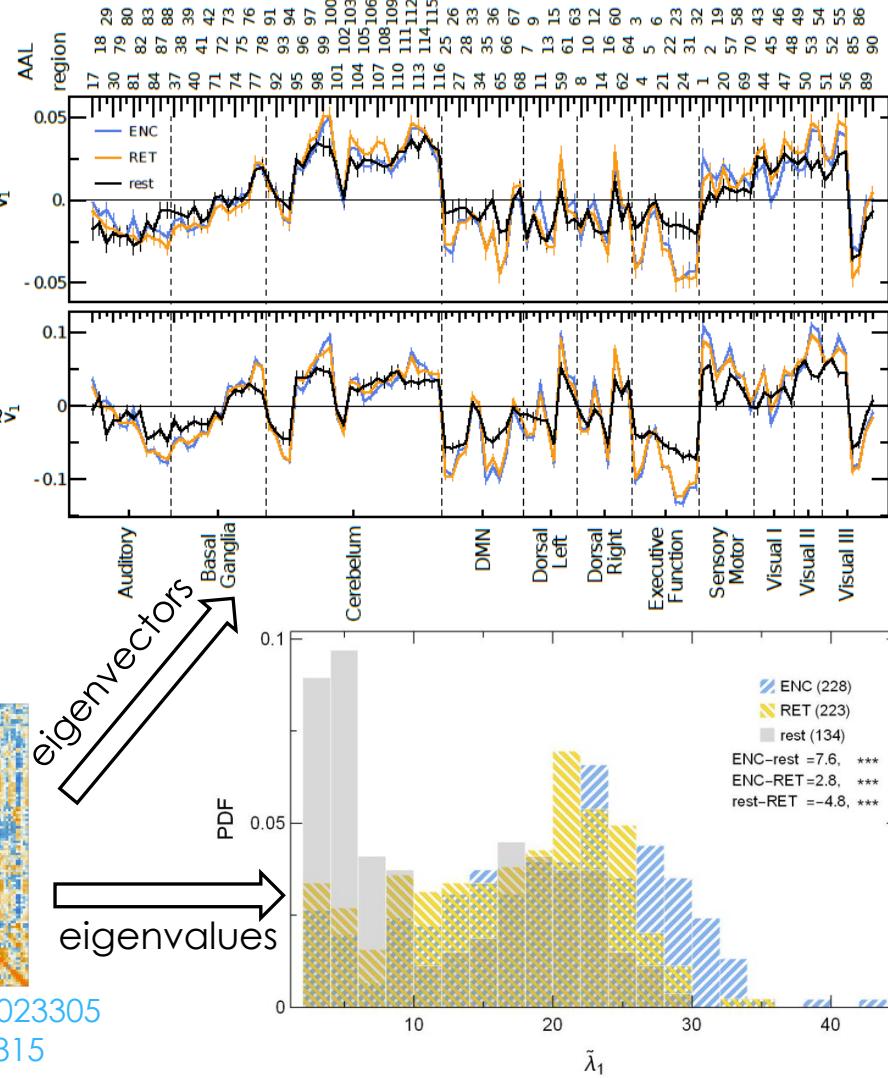
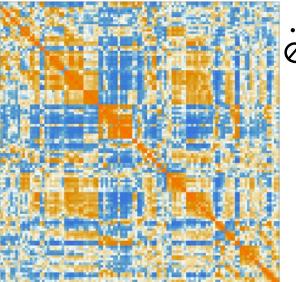
average fluctuation in segments

$$F_{XY}^q(s) = \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} \text{sgn}(F_{XY}^2(\nu, s)) [F_{XY}^2(\nu, s)]^{q/2}$$

correlation matrix

$$\rho(q, s) = \frac{F_{xy}^q(s)}{\sqrt{F_{xx}^q(s) F_{yy}^q(s)}}$$

Time scale dependence  
Fluctuation size dependence



P Oświęcimka et al. (2014). Physical Review E, 89(2):023305  
J Kwapień et al. (2015). Physical Review E, 92(5):052815

# fMRI (functional magnetic resonance imaging )

Data: Working memory task

Technique: Machine Learning

# ENCoding

Was it in the memory set?

# REtrieval

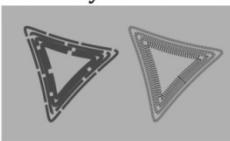
Probe 2000ms

Fixation 450ms



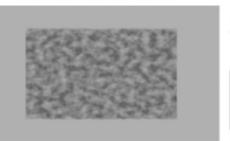
100ms  
Blank

Memory set 1800ms



1000ms  
Blank

Mask 1200ms



2000 -  
16000ms  
Blank

POS



LUR



6000 -  
15000ms  
Blank

NEG



## VISUAL TASKS

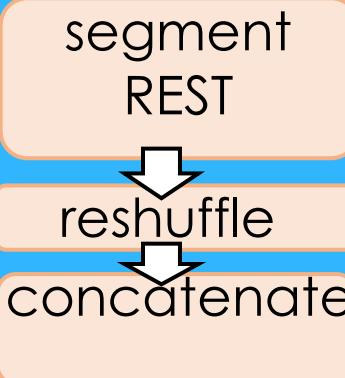
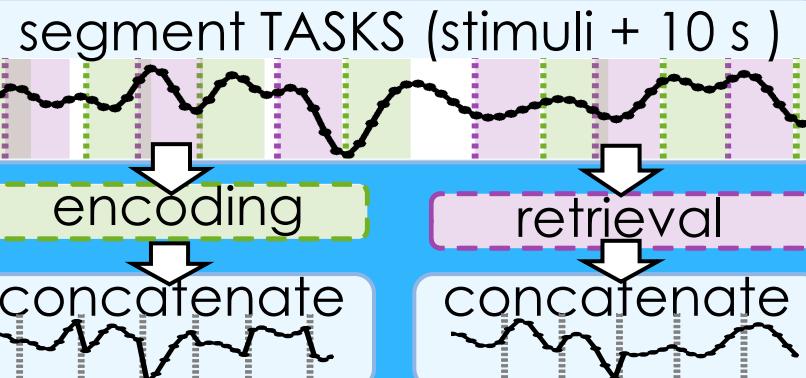
RESTing  
state

verbal

non-verbal



116 ROIs  
(AAL atlas)



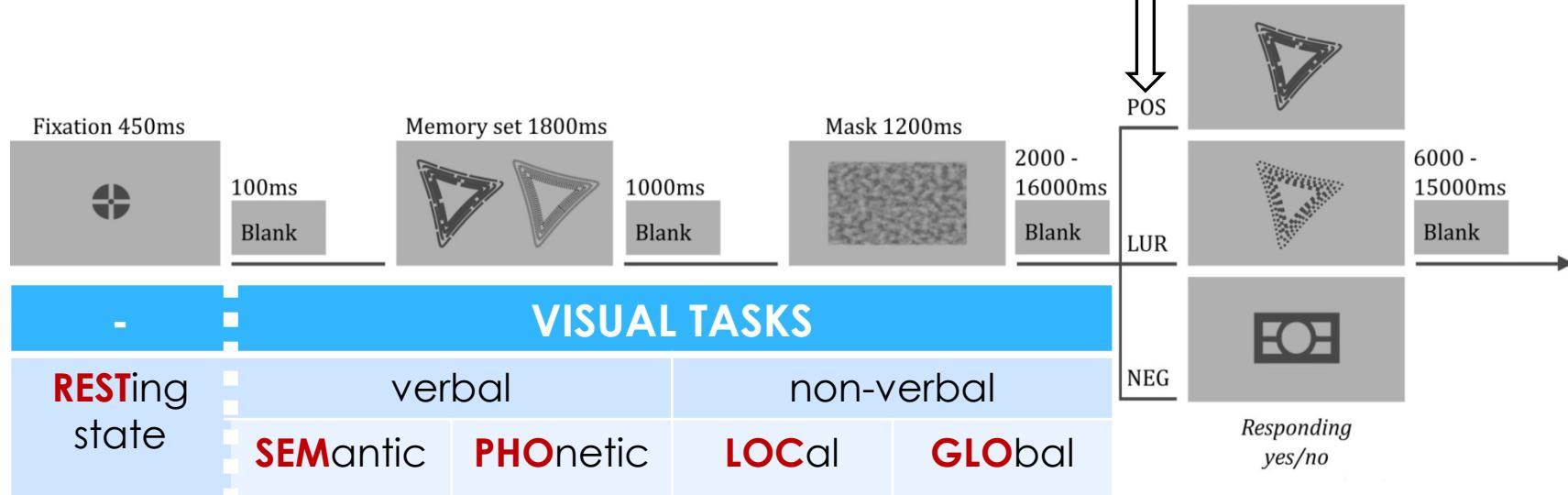
1 time point or  
6 time points  
(NNs)

# ENCoding

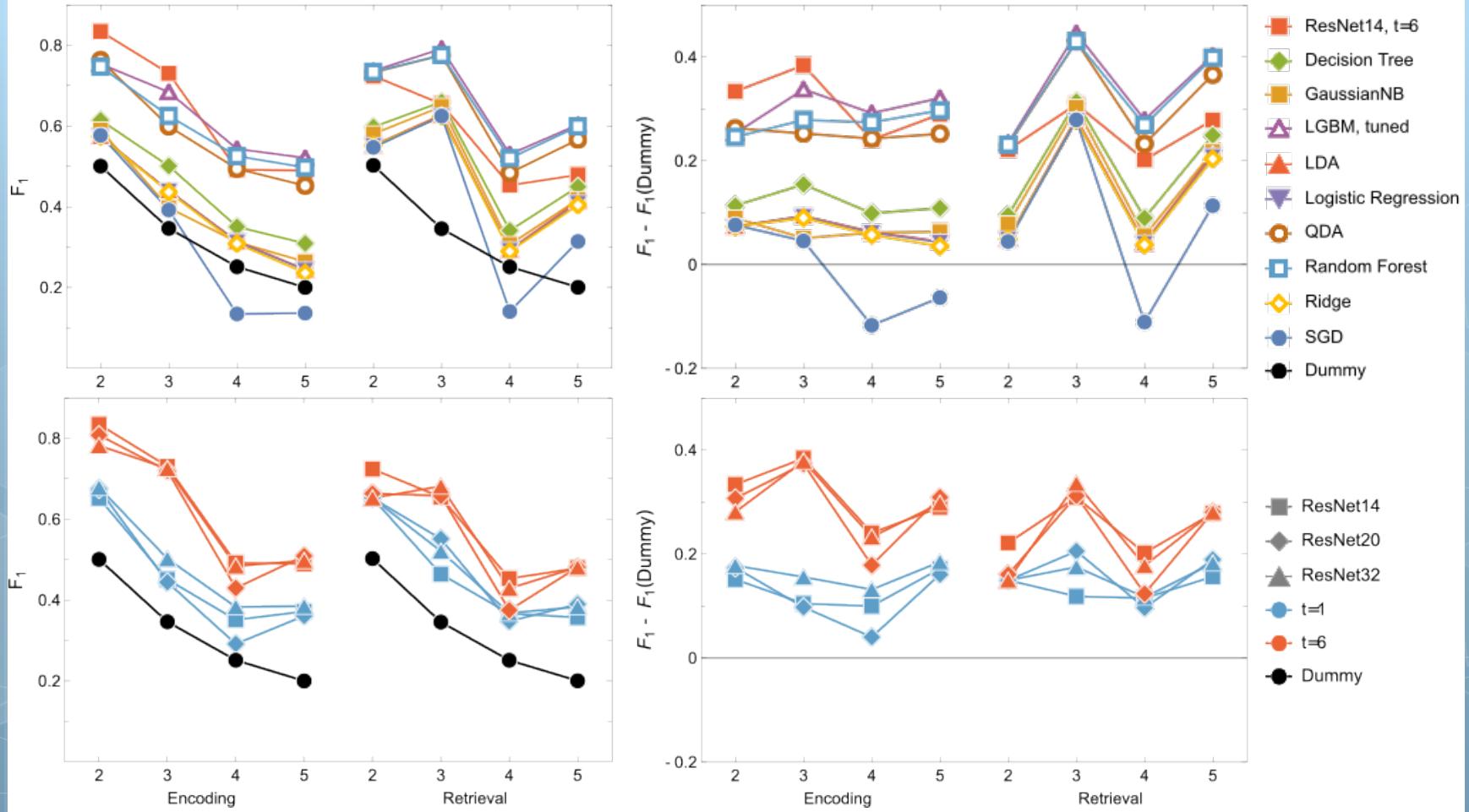
Was it in the memory set?

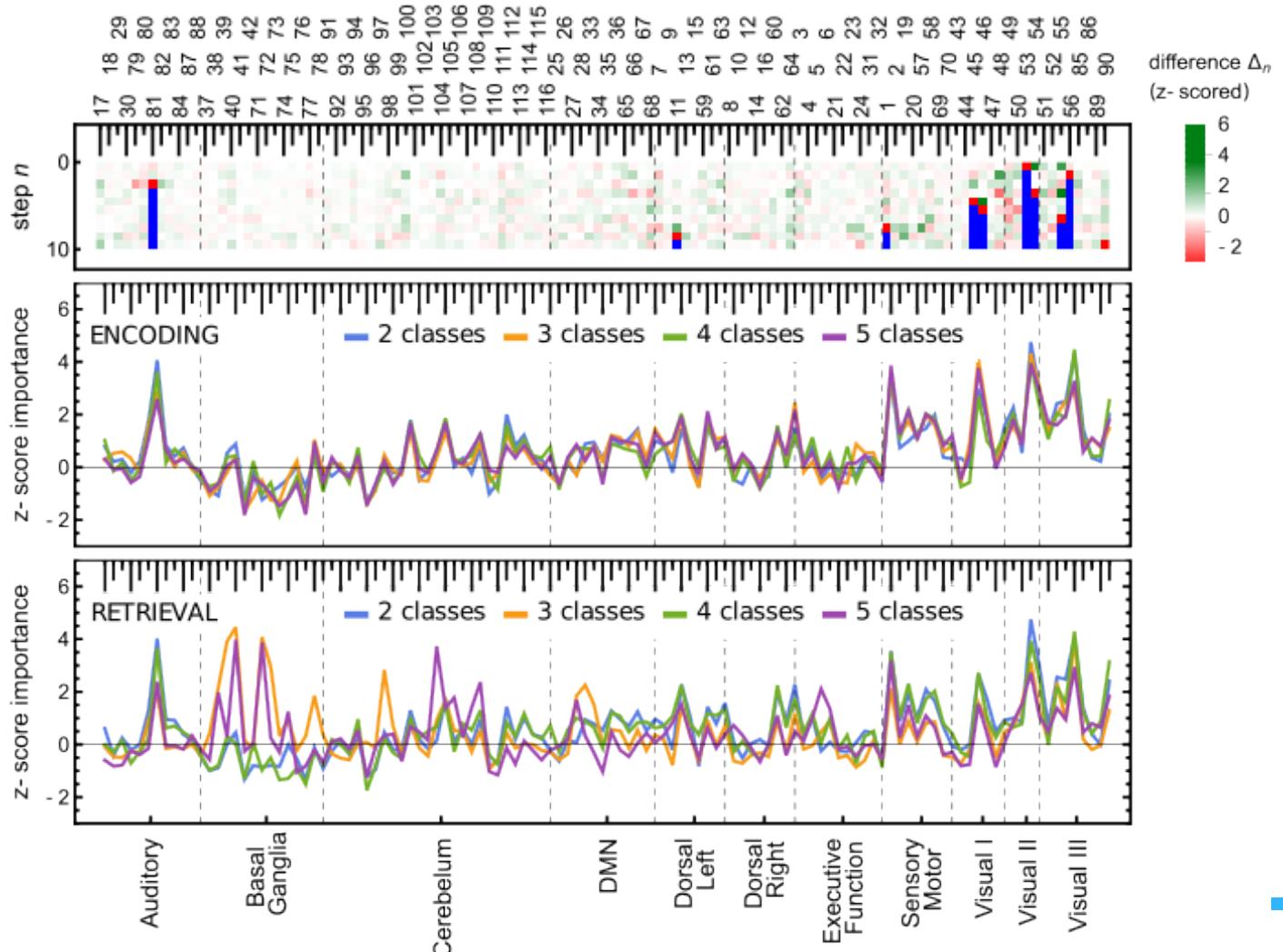
# REtrieval

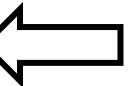
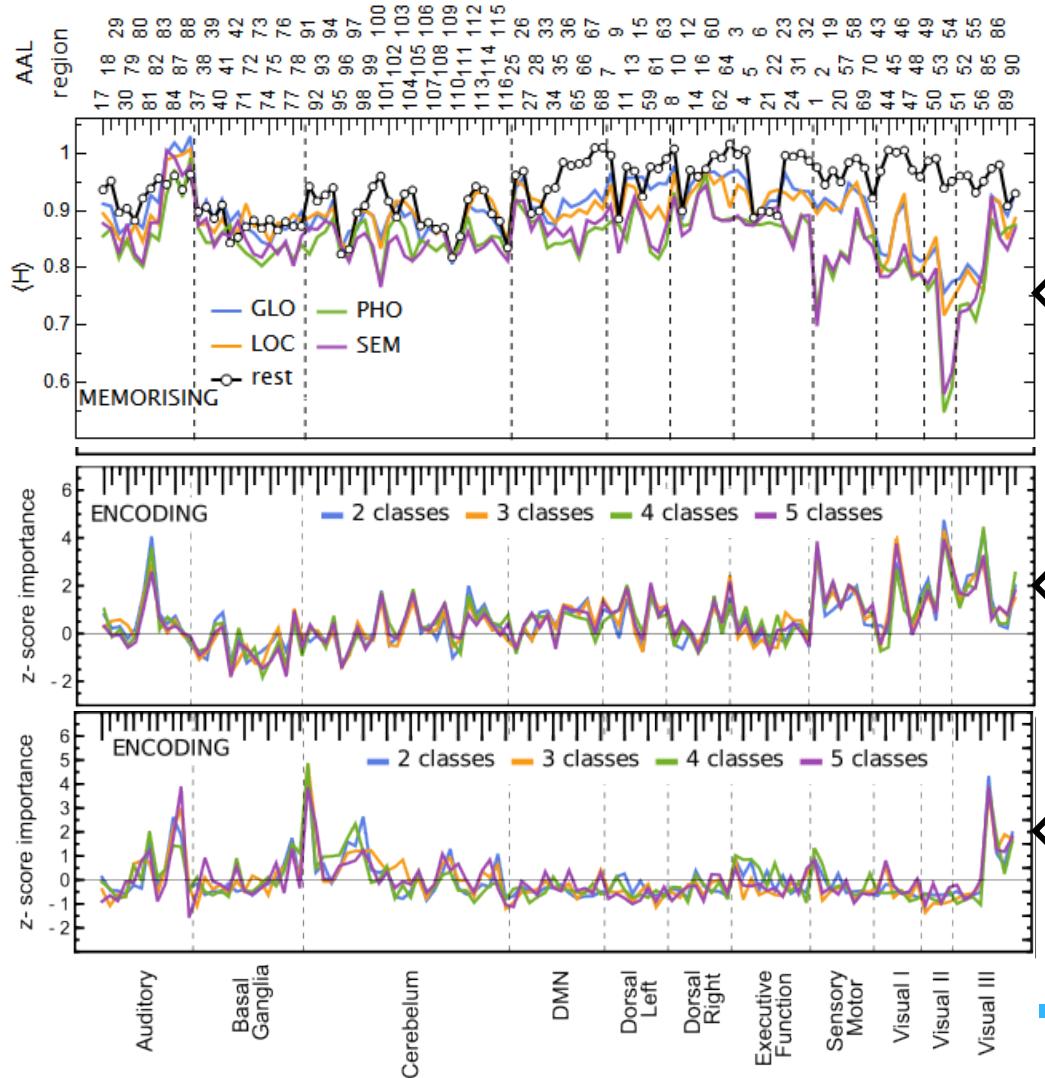
Probe 2000ms



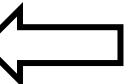
Experiment name	Class 1	Class 2	Class 3	Class 4	Class 5
<b>ENC2</b>	GLO, LOC	SEM, PHO			
<b>ENC3</b>	GLO, LOC	SEM, PHO	REST		
<b>ENC4</b>	GLO	LOC	SEM	PHO	
<b>ENC5</b>	GLO	LOC	SEM	PHO	REST
<b>RET2</b>	GLO, LOC	SEM, PHO			
<b>RET3</b>	GLO, LOC	SEM, PHO	REST		
<b>RET4</b>	GLO	LOC	SEM	PHO	
<b>RET5</b>	GLO	LOC	SEM	PHO	REST



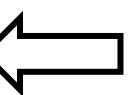




Hurst



LGBM



ResNet

# EEG (electroencephalography)

Data: Multiple-sclerosis

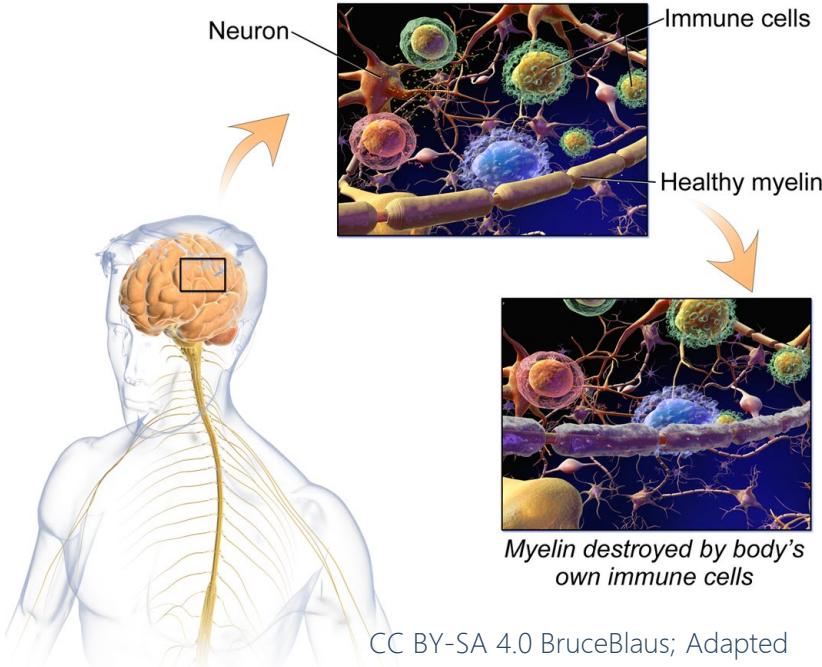
Technique: Multifractality

# Multiple sclerosis

**Multiple sclerosis (MS)** is a chronic immune-mediated disease, the most common nontraumatic disorder of the central nervous system.

Symptoms, depending on the lesion areas, include: fatigue, optic neuritis, depression, heat sensitivity, dizziness, numbness, loss of balance and cognitive dysfunction.

> 2.8 million people with MS worldwide!\*



CC BY-SA 4.0 BruceBlaus; Adapted

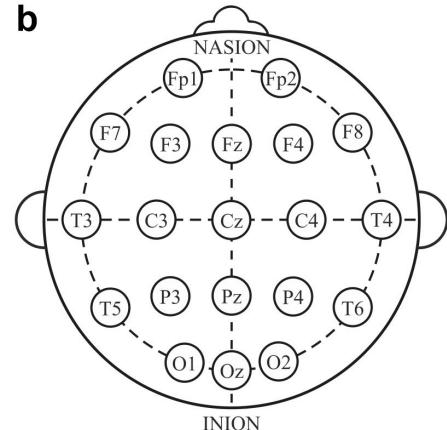
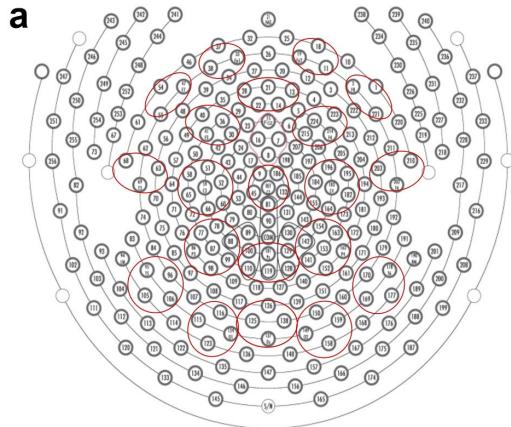
\* C.Walton et al., Mult. Scler. 26(14), 1816 (2020)

# Experiment

- 38 MS patients  
(age:  $34.3 \pm 2.97$ , 19 females)  
from Jagiellonian University's  
Multiple Sclerosis Clinic and  
27 healthy controls  
(age:  $35.6 \pm 2.79$ , 16 females)
- prior to the study, all the  
patients were diagnosed with  
early onset relapsing remitting  
multiple sclerosis (RRMS) with  
**Expanded Disability Status Scale**  
(EDSS) score from 0 to 3.5 points  
(mean:  $1.2 \pm 0.84$ )
- **Aims:** correlates  
of disease duration, EDSS,  
farmacotherapy

# Data

- dense array EEG (256  
electrodes) averaged to 20  
channels
- 1000 Hz sampling rate



# Multifractal DFA

$q$ -dependent fluctuation function

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} [F^2(\nu, s)]^{q/2} \right\}^{1/q}$$

a family of generalised Hurst exponents  $h(q)$ :

$$F_q(s) \sim s^h(q)$$

leading to a multifractal/singularity spectrum of Hölder exponents  $f(\alpha)$  given by

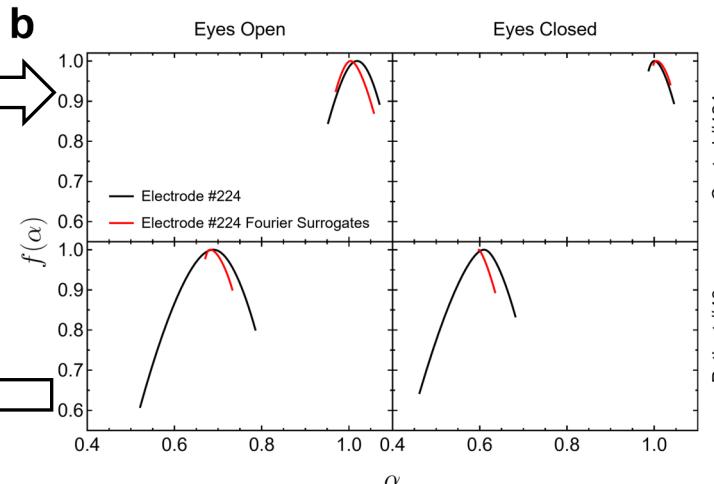
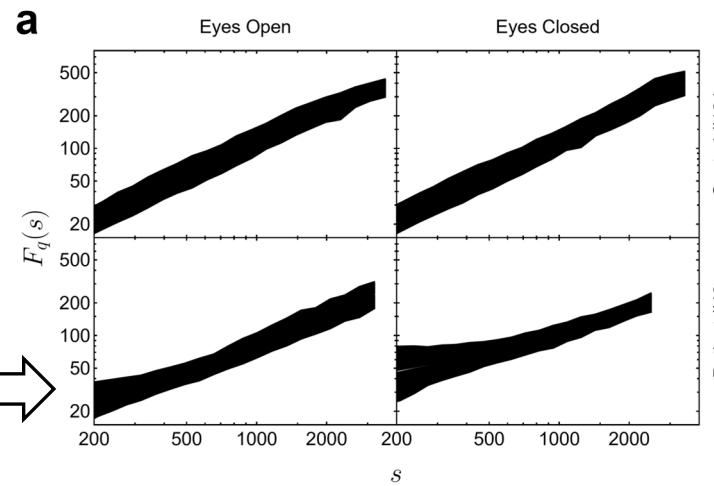
$$\alpha = h(q) + qh'(q)$$

$$f(\alpha) = q(\alpha - h(q)) + 1$$

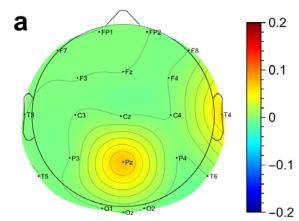
$f(\alpha)$  – a fractal dimension of a subset of the time series with singularities of magnitude  $\alpha$

Spectrum located at:

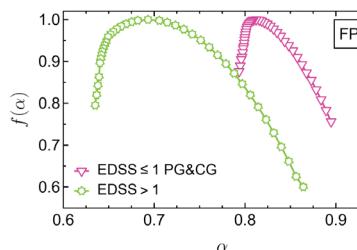
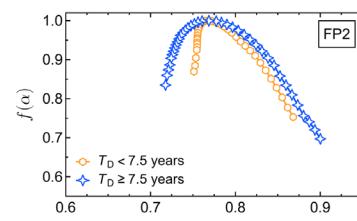
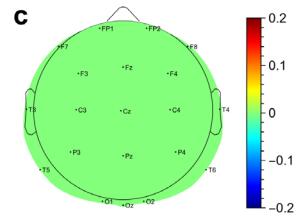
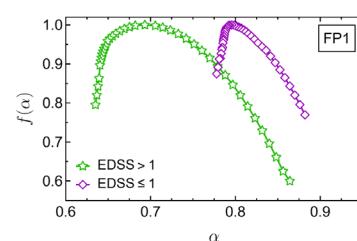
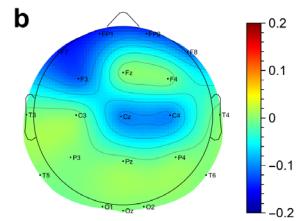
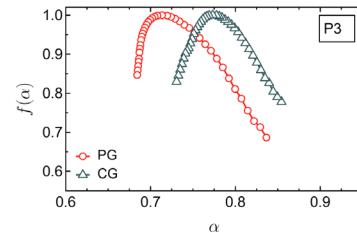
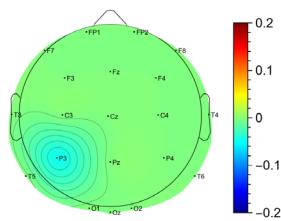
- $\alpha = 0.5$  (weak linear autocorr.)
- $\alpha < 0.5$  (negative autocorrelation)
- $\alpha > 0.5$  (positive autocorrelation)



## Average Hurst Exponents



## Average Multifractal Spectra Width

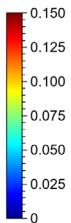
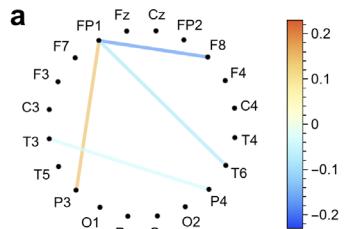


- Control group and patients

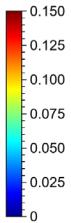
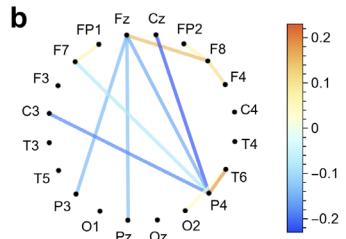
- Patients with EDSS  $> 1$  and patients with EDSS  $\leq 1$

- Patients with the disease duration  $\geq 7.5$  and  $< 7.5$  years

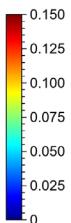
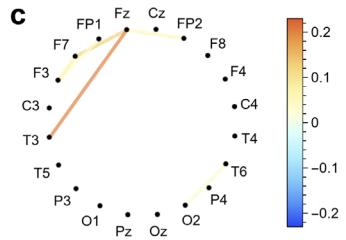
- Patients with EDSS  $> 1$  and the combined group of patients with EDSS  $\leq 1$  and controls.



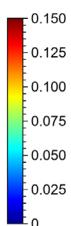
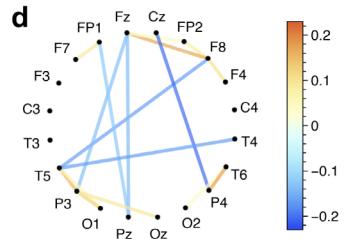
➤ Control group  
and patients



➤ Patients with  
EDSS > 1  
and patients  
with EDSS ≤ 1



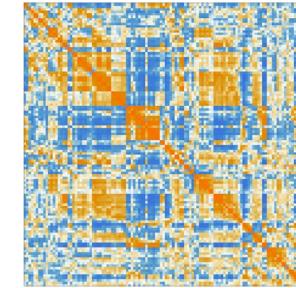
➤ Patients with the  
disease duration  
≥ 7.5 and < 7.5  
years



➤ Patients with  
EDSS > 1 and the  
combined group  
of patients with  
EDSS ≤ 1 and  
controls.

$$\rho(q, s) = \frac{F_{xy}^q(s)}{\sqrt{F_{xx}^q(s)F_{yy}^q(s)}}$$

↑ Time scale dependence  
↑ Fluctuation size dependence



# Conclusions

Hurst exponents of fMRI signals and their detrended cross-correlations:

- are sensitive to the type of task the brain is processing
- are different for people with and without cognitive dysfunction (plus some local differences in multifractal spectra)



T Marek  
M Fajfrowicz  
H Ogińska  
M Gawłowska



P Oświęcimka  
M Wątorek  
W Tomczyk  
M Tutajewski



B Sikora-Wachowicz  
K Lewandowska  
A Ceglarek  
N Golonka  
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